Electric Circuits. Nodal and Mesh Analysis

Theory and solved problems

Alfonso Bachiller Soler
Ramón Cano González
Miguel Ángel González Cagigal
Department of Electrical Engineering
Universidad de Sevilla

DIAZ DE SANTOS

Contents

1 Introduction

A circuit can be defined as an ensemble of electrical elements interconnected with each other. In general terms, the resolution of an electrical circuit is based on the calculation of the voltages and currents for all those elements, given that, using the mentioned magnitudes, other variables can be easily obtained, as might be the power or the energy.

In any electrical circuit, Kirchhoff's laws must be satisfied jointly with the defining equation of the involved elements.

Because of the interconnection of the different elements, Kirchhoff's current law must be satisfied in each node [1] of the circuit. Likewise, Kirchhoff's voltage law is satisfied in each closed circuit path (loop or mesh). Finally, voltage and current of each element in the circuit are related through the corresponding defining equation.

An analysis will be subsequently made on the number of equations and unknowns involved in the calculation of the voltages and currents for the elements of a circuit. In this regard, let be n and c the number of nodes and of two-terminal elements respectively, leading to $2c$ unknowns (the corresponding voltages and currents) to be determined. On the other hand, the following linearly-independent equations can be formulated:

- Kirchhoff's current law: $n - 1$.
- Kirchhoff's voltage law: $c - n + 1$ (number of meshes).
- Defining equations of the different elements: c.

It can be noticed that number of unknowns matches the number of equations. In the previous analysis, the reduction in the number of unknowns (and consequently equations) derived from the dependent electrical sources has not been taken into account.

[1] A node is defined as a point where two or more elements of a circuit are connected.

Example 1.0.1. In the circuit represented in Fig. 1.1, obtain the required equations for the calculation of the voltages and currents of every element. The nodes and closed circuit paths have been remarked, where the corresponding Kirchhoff's laws will be applied.

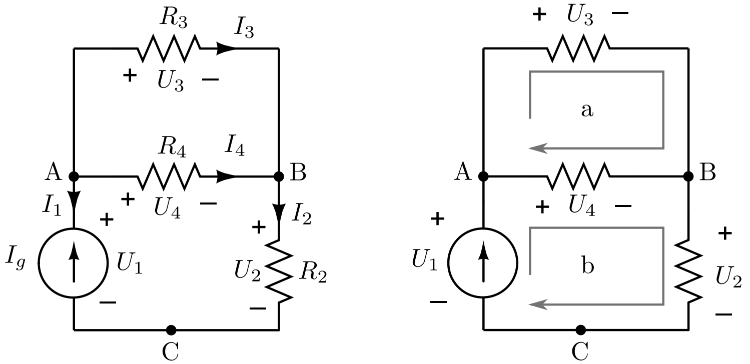

Figure 1.1.

Solution. It can be noticed that the circuit has 3 nodes and 4 elements, meaning:

$$n = 3 \; ; \; c = 4$$

In this particular case, the number of unknowns is 8 ($2c{=}2 \cdot 4{=}8$):

$$U_1, U_2, U_3, U_4, I_1, I_2, I_3, I_4$$

Regarding the equations:

- Kirchhoff's current law: $n - 1{=}3 - 1{=}2$

$$\begin{aligned} \text{Node A}: & \; -I_1 - I_3 - I_4 = 0 \\ \text{Node B}: & \; I_3 + I_4 - I_2 = 0 \end{aligned}$$

  It can be noticed that the application of Kirchhoff's current law in node C leads to an equation dependent on the previous two, providing no additional information.

- Kirchhoff's voltage law: $c - n + 1{=}4 - 3 + 1{=}2$

$$\begin{aligned} \text{Mesh a}: & \; U_3 - U_4 = 0 \\ \text{Mesh b}: & \; U_4 + U_2 - U_1 = 0 \end{aligned}$$

- Defining equations of the elements: $c = 4$

$$\begin{aligned} I_1 &= -I_g \\ U_2 &= R_2 I_2 \\ U_3 &= R_3 I_3 \\ U_4 &= R_4 I_4 \end{aligned}$$

Rearranging the equations, the following system is obtained in matrix form:

$$\begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -R_2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -R_3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -R_4 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -I_g \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The previous process can be applied for relatively simple circuits. However, as the complexity increases, the excessive number of elements makes it necessary to use alternative analysis techniques in order to simplify the resolution. The system of $2c$ equations can be reduced by choosing an adequate set of "basic" variables, from which any other magnitude can be calculated. For example, if the considered basic variables are the node voltages, the **nodal analysis method** arises, while, if the mesh currents are selected, the **mesh analysis method** is formulated. Using these techniques, the circuit resolution is made with a minimum number of equations.

In the next two chapters, the mentioned methods will be analyzed, and diverse problems will be solved representing the different case studies which can be observed in practice.

Once both analysis methods are presented, an important issue is the appropriate election of the most efficient technique in each case. In general terms, the correct answer is determined by the following factors:

1. Circuit nature

- Circuits including elements in series and voltage sources: **Meshes**.
- Circuits including elements in parallel and current sources: **Nodes**.
- Circuits with less nodes than meshes: **Nodes**.
- Circuits with more nodes than meshes: **Meshes**.
- Circuits with a non-planar topology: **Nodes**.

2. Required information

- If node voltages are requested: **Nodes**.
- If mesh currents are requested: **Meshes**.

Nevertheless, it should be remarked that the nodal analysis method is easier to program in a computer. As a curiosity, the method based on node voltages is used for the resolution of load flows in electric power systems.